

alpha-like four-nucleon correlations

M. Hasegawa and K. Kaneko

Laboratory of Physics, Fukuoka Dental College, Fukuoka 814-0193, Japan

Department of Physics, Kyushu Sangyo University, Fukuoka 813-8503, Japan

(July 6, 1999)

Abstract

An extended pairing plus QQ force model, which has been shown to successfully explain the nuclear binding energy and related quantities such as the symmetry energy, is applied to study the alpha-like four-nucleon correlations in $1f_{7/2}$ shell nuclei. The double difference of binding energies, which displays a characteristic behavior at $N \approx Z$, is interpreted in terms of the alpha-like correlations. Important roles of proton-neutron interactions forming the alpha-like correlated structure are discussed.

21.10Dr;21.60-n;21.60Cs;21.60.Gx

Typeset using REVTeX

Recently, the present authors [1,2] have proposed a schematic interaction for $N \approx Z$ nuclei in $1f_{7/2}$ and $1g_{9/2}$ shell regions and have applied it to explain double differences of binding energies discussed in Refs. [3,4]. The double difference of binding energies of neighboring even-even nuclei was discussed to investigate α -like four-nucleon correlations in early work [5,6]. The calculations in Refs. [1,2] showed that the schematic interaction is applicable to the $1f_{7/2}$ shell nuclei where the α -like four-nucleon correlations are very important [7–12]. Our interaction, which is composed of the isospin-invariant monopole pairing (P_0), quadrupole pairing (P_2), quadrupole-quadrupole (QQ) forces and J -independent isoscalar proton-neutron (p - n) force ($V_{\pi\nu}^{\tau=0}$), is useful to study the roles of interactions. The purpose of this note is to investigate cooperation of the $P_0 + P_2 + QQ$ force and $V_{\pi\nu}^{\tau=0}$, or competition of the p - n and like-nucleon (p - p , n - n) interactions, in the α -like four-nucleon correlations.

It is well known that the single j space $(f_{7/2})^n$ is a reasonable model space for the $1f_{7/2}$ shell nuclei [13]. We adopted the $(f_{7/2})^n$ model with the $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction in Refs. [1,2]. Our effective Hamiltonian which describes valence nucleons outside the doubly-closed-shell core ^{40}Ca is given by

$$H = \epsilon_{7/2} \hat{n} + V_{PQ} + V_{\pi\nu}^{\tau=0}, \quad (1)$$

$$\begin{aligned} V_{PQ} &= V(P_0) + V(P_2) + V(QQ) \\ &= -g_0(2j+1) \sum_{\kappa} A_{001\kappa}^{\dagger} A_{001\kappa} - G_2 \sum_{M\kappa} A_{2M1\kappa}^{\dagger} A_{2M1\kappa} \\ &\quad - X \sum_{J(\tau)} (-)^{J+1} 5W(jjjj : 2J) \sum_{M\kappa} A_{JM\tau\kappa}^{\dagger} A_{JM\tau\kappa}, \end{aligned} \quad (2)$$

$$V_{\pi\nu}^{\tau=0} = -k^0 \sum_{J=\text{odd}} \sum_M A_{JM00}^{\dagger} A_{JM00}, \quad (3)$$

with

$$A_{JM\tau\kappa}^{\dagger} = \sum_{mm'} \langle jmjm' | JM \rangle \sum_{\rho\rho'} \langle \frac{1}{2}\rho \frac{1}{2}\rho' | \tau\kappa \rangle \frac{1}{\sqrt{2}} c_{m\rho}^{\dagger} c_{m'\rho'}^{\dagger},$$

where $W(jjjj : 2J)$ is the Racah coefficient. The interaction has four parameters g_0 , G_2 , X and k^0 .

Although the Coulomb energy cancels out in the double differences of binding energies, it must be appropriately evaluated in order to compare calculated energies of Hamiltonian

(1) with observed binding energies. We evaluate experimental energies of the ground states as follows:

$$W_0(Z, N) = B(Z, N) - B(^{40}\text{Ca}) - \lambda(A - 40) - \Delta E_C(n_p, n_n), \quad (4)$$

where $B(Z, N)$ is the nuclear binding energy, λ is the base level of the single-particle state energy $\epsilon_{7/2}$ and $\Delta E_C(n_p, n_n)$ is the Coulomb energy correction for the valence nucleons $n = n_p + n_n$ ($n_p = Z - 20$ and $n_n = N - 20$). Following Caurier *et al.* [14], we evaluate $\Delta E_C(n_p, n_n)$ by the function

$$\Delta E_C(n_p, n_n) = 7.279 n_p + 0.15 n_p(n_p - 1) - 0.065 n_p n_n. \quad (5)$$

We fix $\lambda = -8.364$ MeV so that $W_0(^{41}\text{Ca}) = W_0(^{40}\text{Ca}) = 0.0$, namely $\epsilon_{7/2} = 0.0$.

We use the constant force strengths $g_0=0.59$, $G_2=0.9$ and $X=1.2$ in MeV neglecting A -dependence for the $P_0 + P_2 + QQ$ force and put A -dependence for the p - n force $V_{\pi\nu}^{\tau=0}$ as $k^0 = 1.9 \times (48/A)$ MeV. These parameters reproduce well the Coulomb corrected energies of valence nucleons near ^{48}Cr [1], and the double differences of binding energies for the $1f_{7/2}$ shell nuclei, especially the peak value at $N=Z$ [2]. These results tell us that our model is reliable enough for our purpose to analyze respective contributions of interactions in the α -like correlations.

According to Ref. [11], the ground states of $(f_{7/2})^{4m}$ systems with $n_p=n_n=2m$ can be approximated by condensed states of the $I=T=0$ α -like clusters (2p-2n units)

$$\begin{aligned} |(f_{7/2})^{4m}: \text{gr}, I = T = 0\rangle &\approx \frac{1}{\sqrt{N_0}} (\alpha_0^\dagger)^m |A_0\rangle, \\ \alpha_0^\dagger &= \sum_{J\tau} \Psi^{(0)}(J\tau, J\tau : I = T = 0) (A_{J\tau}^\dagger A_{J\tau}^\dagger)_{I=0, T=0}, \end{aligned} \quad (6)$$

where N_0 is a normalization constant and $|A_0\rangle$ is the doubly-closed-shell core state. The approximate treatment of the $(\alpha_0)^m$ model gives the energy -32.04 MeV to the exact energy -32.20 MeV for the $(f_{7/2})^8$ system. The modified treatment which takes the contributions of 2p-2n units with $I > 0$ into account reproduces the energy -32.20 MeV. Therefore, the $I=T=0$ ground state of the $(f_{7/2})^8$ system is described as the dominant state $(\alpha_0)^2$

supplemented with other components of four-nucleon units $(\alpha_{I>0,T=0})^2$. (We showed in Ref. [11] that the ^{48}Cr ground state is approximately described in terms of $\sum_I(\alpha_{I>0,T=0})^2$ in the full fp space.) The squared amplitude of the dominant component $(\alpha_0)^2$ is 0.973 in the present calculation. The approximate treatment of the $(\alpha_0)^3$ model gives -52.14 MeV to the exact energy -52.39 MeV for the $(f_{7/2})^{12}$ system. This result supports the goodness of our picture (6). The shell model calculations for the odd-odd systems $(f_{7/2})^{n=6,10}$ tell us that their ground states can be approximated by

$$|(f_{7/2})^{4m+2}: \text{gr}, I = 0, T = 1\rangle \approx \frac{1}{\sqrt{N_1}} A_{001\kappa}^\dagger |(f_{7/2})^{4m}: \text{gr}, I = T = 0\rangle, \quad (7)$$

where N_1 is a normalization constant. The overlap of the approximate state (7) with the exact state is 0.983 for the $(f_{7/2})^6$ system and 0.987 for the $(f_{7/2})^{10}$ system. The approximation (7) therefore holds well. We can say that the α -like four-nucleon correlations play a dominant role in the $(f_{7/2})^{4m}$ systems with $n_p=n_n=2m$ and the $\tau=1$ monopole pairing correlation is also important when an additional proton-neutron pair exists. Based on this knowledge, let us analyze competition of correlations.

We calculated interaction energies of the $P_0 + P_2 + QQ$ force and $V_{\pi\nu}^{\tau=0}$ separately. The calculated results for the isotopes with $n_p=0, 2$ and 4 , which correspond to the Ca, Ti and Cr isotopes, are shown in Fig. 1. Look at the expectation value of the $P_0 + P_2 + QQ$ force for the ground states of the $n_p=2$ isotopes. As the neutron number n_n increases, the energy gain increases more rapidly than that of the $n_p=0$ isotopes up to $n_n = 2$ and the two lines of $n_p=0$ and $n_p=2$ become nearly parallel for $n_n \geq 2$. The line of $n_p=4$, which is parallel to that of $n_p=2$ up to $n_n=2$, shows an additional energy gain at $n_n=n_p=4$ and becomes again parallel to that of $n_p=2$ for $n_p \geq 4$. We know that a considerably large energy gain of the $P_0 + P_2 + QQ$ force is attained every time when a 2p-2n unit with $T=0$ is formed. At the points $n_n=n_p=2$ and 4 , in Fig. 1, the energy gain of $V_{\pi\nu}^{\tau=0}$ also shows a peak (see the bend in the line of $\langle V_{\pi\nu}^{\tau=0} \rangle$). The ground states of the $(f_{7/2})^{4m}$ systems with $n_n=n_p=2m$ have the α -like correlated structure as mentioned above. Therefore, the large energy gain of the $P_0 + P_2 + QQ$ force at $n_n=n_p=2m$ drives the system to form the α -like units. It should

be remembered here that the ground-state wavefunction is determined by the $P_0 + P_2 + QQ$ force [1]. The p - n interaction $V_{\pi\nu}^{\tau=0}$ which is written as $-\frac{1}{2}k^0\{\frac{\hat{n}}{2}(\frac{\hat{n}}{2} + 1) - \hat{T}^2\}$ endows the $T = 0$ α -like correlated state with a very large energy gain.

To discuss the large binding energies of the $n_n=n_p=2m$ systems due to the α -like four-nucleon correlations, Gambhir, Ring and Schuck [6] considered differences of binding energies

$$S(A_0 + 4m + 2) = B(Z_0 + 2m, N_0 + 2m) - \frac{1}{2}\{B(Z_0 + 2m + 2, N_0 + 2m) + B(Z_0 + 2m, N_0 + 2m + 2)\}, \quad (8)$$

$$S(A_0 + 4m + 4) = \frac{1}{2}\{B(Z_0 + 2m + 2, N_0 + 2m) + B(Z_0 + 2m, N_0 + 2m + 2)\} - B(Z_0 + 2m + 2, N_0 + 2m + 2). \quad (9)$$

They explained in terms of the α -like correlated units that $S(A_0 + 4m + 4)$ is systematically larger than $S(A_0 + 4m + 2)$. The two quantities $S(A_0 + 4m + 4)$ and $S(A_0 + 4m + 2)$ have contributions from the Coulomb energy. If we consider the difference $S(A_0 + 4m + 4) - S(A_0 + 4m + 2)$, the contributions from the Coulomb energy and single-particle energy nearly disappear (though a small effect of the Coulomb energy remains as long as we adhere the approximation (5)). It is convenient to take up the Coulomb corrected energy of valence nucleons $W_0(Z, N)$ instead of $B(Z, N)$, because the effective Hamiltonian aims to reproduce $W_0(Z, N)$. We are considering the $N \approx Z$ nuclei in which protons and neutrons occupy the same shell. The ground states of the $A_0 + 4m + 2$ systems are the $I=0, T=1$ states (in the $f_{7/2}$ shell nuclei) and the Hamiltonian is isospin invariant. Hence, we have the approximate relations

$$W_0(Z_0 + 2m + 2, N_0 + 2m) \approx W_0(Z_0 + 2m, N_0 + 2m + 2) \approx W_0(Z_0 + 2m + 1, N_0 + 2m + 1). \quad (10)$$

Let us define the following quantities corresponding to Eqs. (8) and (9):

$$S'(n_p + 2, n_n) = W_0(n_p, n_n) - W_0(n_p + 2, n_n), \quad (11)$$

$$S'(n_p + 2, n_n + 2) = W_0(n_p, n_n + 2) - W_0(n_p + 2, n_n + 2), \quad (12)$$

When neglecting the small Coulomb energy effect, we can expect

$$\begin{aligned}\delta S'(n_p + 2, n_n + 2) &= S'(n_p + 2, n_n + 2) - S'(n_p + 2, n_n) \\ &\approx S(A_0 + 4m + 4) - S(A_0 + 4m + 2).\end{aligned}\tag{13}$$

The difference $\delta S'(n_p + 2, n_n + 2) = S'(n_p + 2, n_n + 2) - S'(n_p + 2, n_n)$ represents the correlations between two proton and two neutrons from the viewpoint of Gambhir, Ring and Schuck [6]. The same quantity (which is called the double difference of binding energies) has recently been investigated as the p - n interaction in Refs. [3,4], where the remarkable increase of this quantity at $N=Z$ (or $n_n = n_p$) is also discussed.

The quantity $S'(n_p + 2, n)$ corresponds to $(A_n - B_n) + (a_n - b_n)$ and $(B_n - C_n) + (b_n - c_n)$ in Fig. 1. For instance, $S'(4, 4)$ is shown by the two solid lines connecting B_n and C_n (b_n and c_n) at $n_n=4$, and $S'(4, 2)$ is shown by the two broken lines at $n_n=2$. Similarly, $S'(2, 2)$ and $S'(2, 0)$ are shown by the solid and broken lines connecting A_n and B_n (a_n and b_n) at $n_n=2$ and 0, respectively. The solid line representing $S'(2m + 2, 2m + 2)$ is longer than the broken line representing $S'(2m + 2, 2m)$ both in $\langle P_0 + P_2 + QQ \rangle$ and $\langle V_{\pi\nu}^{\tau=0} \rangle$. This was discussed as the evidence of the α -like four-nucleon correlations by Gambhir, Ring and Schuck [6]. They, however, did not consider the p - n interaction like $V_{\pi\nu}^{\tau=0}$ and hence could not reproduce sufficient values for $S(A_0 + 4m + 4) - S(A_0 + 4m + 2)$. We know the indispensable role of $V_{\pi\nu}^{\tau=0}$ in the α -like four-nucleon correlations.

Thus, the double difference $\delta S'(n_p + 2, n_n + 2)$ is a good measure of the α -like correlation energy. We showed in Ref. [2] that $\delta S'$ has a triangular-shape peak at $N = Z$. This can be visually understood in Fig. 1 as follows: The differences $(b_n - c_n) - (b_{n-2} - c_{n-2})$ and $(B_n - C_n) - (B_{n-2} - C_{n-2})$ have maximum values at $n_n=4$ and are secondly large at $n_n=3$ and 5. The quantity $(B_n - C_n) - (B_{n-2} - C_{n-2})$ becomes negligible when $|n_n - n_p| > 1$, since the two lines of $n_p=2$ and 4 for $\langle P_0 + P_2 + QQ \rangle$ are nearly parallel when $n_n \leq 2$ and $n_n \geq 4$, namely $\delta S'$ approximately comes from $V_{\pi\nu}^{\tau=0}$ and becomes roughly a constant $k^0/2$ for $|n_n - n_p| > 1$.

To analyze the interesting behavior of the p - n interaction (correlations between two

protons and two neutrons) in detail, let us now see the contributions of p - n and like-nucleon (p - p and n - n) correlations separately. Since we know the behavior of the p - n interaction $V_{\pi\nu}^{\tau=0}$, we show the two types of contributions from the $P_0 + P_2 + QQ$ force in Fig. 2. This figure demonstrates that the p - n interaction of the $P_0 + P_2 + QQ$ force gives energy gains roughly proportional to the neutron number till $n_n=n_p$, and attains a maximum energy gain when a maximum number of α -like units are formed at $n_n=n_p=2m$. Once all the protons join the α -like units, additional neutrons interact weakly with the protons in the α -like units through the p - n interaction of the $P_0 + P_2 + QQ$ force (somewhat decreasing the energy gain attained by forming the α -like units). On the other hand, the n - n correlations between the additional neutrons are still observed in Fig. 2.

Accordingly, we reach the following conclusion. The remarkable peak observed at $N = Z$ in the double difference of binding energies is closely related to the α -like four-nucleon correlations. The maximum contribution of the p - n interaction in cooperation with the like-nucleon interactions within the $P_0 + P_2 + QQ$ force, forming the α -like correlated structure, attains a special energy gain in the $N=Z$ even-even systems. Another p - n interaction $V_{\pi\nu}^{\tau=0}$ makes the structure stable by endowing a very large energy gain. It is interesting that the coupling between the α -like units and residual neutrons is weak in the $P_0 + P_2 + QQ$ force. This suggests a direct product state of the α -like correlated structure and residual pairing correlated neutrons at the end. In the $N=Z$ odd-odd systems, the $\tau=1$ monopole pairing interaction is important for the last p - n pair and the wavefunction is approximated by Eq. (7). We have made our study within the $(f_{7/2})^n$ model, in this note. The collective feature of the α -like four-nucleon correlations over many j shells could affect the binding energy little but may considerably change the detailed structure. Still, the outline of our interpretation about the p - n interactions, the double difference of binding energies and the α -like correlations using the $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction will remain valid. The α -like correlations should be stressed even in the shell model approach to the fp shell nuclei.

REFERENCES

- [1] M. Hasegawa and K. Kaneko, Phys. Rev. **C59**, 1449 (1999).
- [2] K. Kaneko and M. Hasegawa, Phys. Rev. **C60**, (1999).
- [3] J. -Y. Zhang, R. F. Casten, and D. S. Brenner, Phys. Lett. **B227**, 1 (1989).
- [4] D. S. Brenner, C. Wesselborg, R. F. Casten, D. D. Warner and J.-Y. Zhang, Phys. Lett. **B243**, 1 (1990).
- [5] G. G. Dussel, R. J. Liotta, and R. P. J. Perazzo, Nucl. Phys. **A388**, 606 (1982).
- [6] Y.K. Gambhir, P. Ring and P. Schuck, Phys. Rev. Lett. **51**, 1235 (1983).
- [7] F. Michel, G. Reidemeister and S. Ohkubo, Phys. Rev. **C34**, 1248 (1986); Phys. Rev. Lett. **57**, 1215 (1986); Phys. Rev. **C37**, 292 (1988).
- [8] T. Wada and H. Horiuchi, Phys. Rev. **C38**, 2063 (1988).
- [9] S. Ohkubo, Phys. Rev. **C38**, 2377 (1988).
- [10] T. Yamaya and S. Oh-ami, M. Fujiwara, T. Itahashi, K. Katori, M. Tosaki, S. Kato, S. Hatori and S. Ohkubo, Phys. Rev. **C42**, 1935 (1990).
- [11] M. Hasegawa, S. Tazaki and R. Okamoto, Nucl. Phys. **A592**, 45 (1995).
M. Hasegawa and S. Tazaki, Nucl. Phys. **A633**, 266 (1998).
- [12] S. Ohkubo et al., Prog. Theor. Phys. Suppl. No. **132** (1998).
- [13] J. D. McCullen, B. F. Bayman and L. Zamick, Phys. Rev. **4**, B515 (1964).
- [14] E. Caurier, A.P. Zuker, A. Poves and G. Martínez-Pinedo, Phys. Rev. **C50**, 225 (1994).

FIGURES

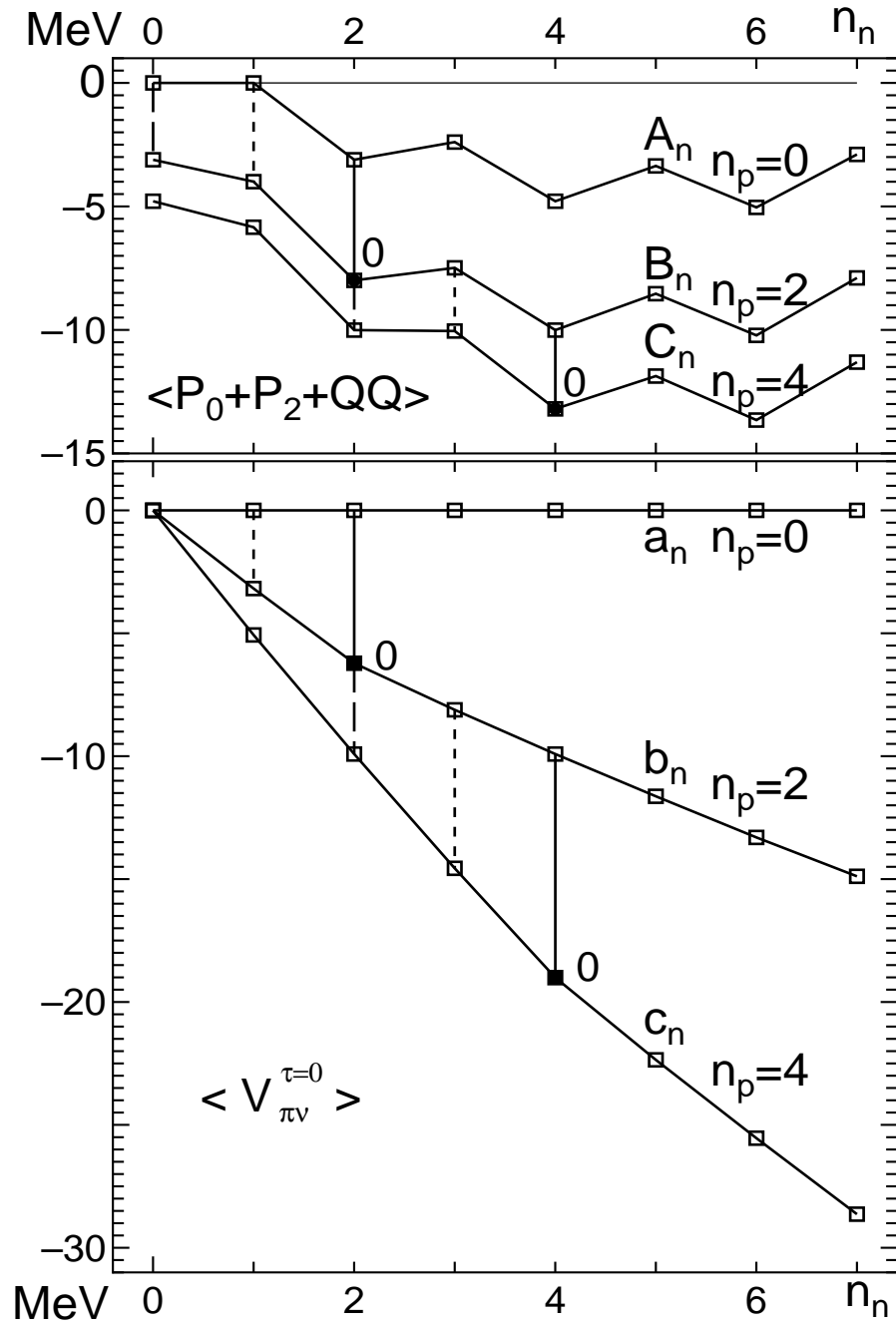


FIG. 1. Interaction energies of the $P_0 + P_2 + QQ$ force and $V_{\pi\nu}^{\tau=0}$ in the $n_p=0, 2$ and 4 isotopes. The states with $T=0$ are shown by the solid squares. The isospin increases as aparting from the solid squares.

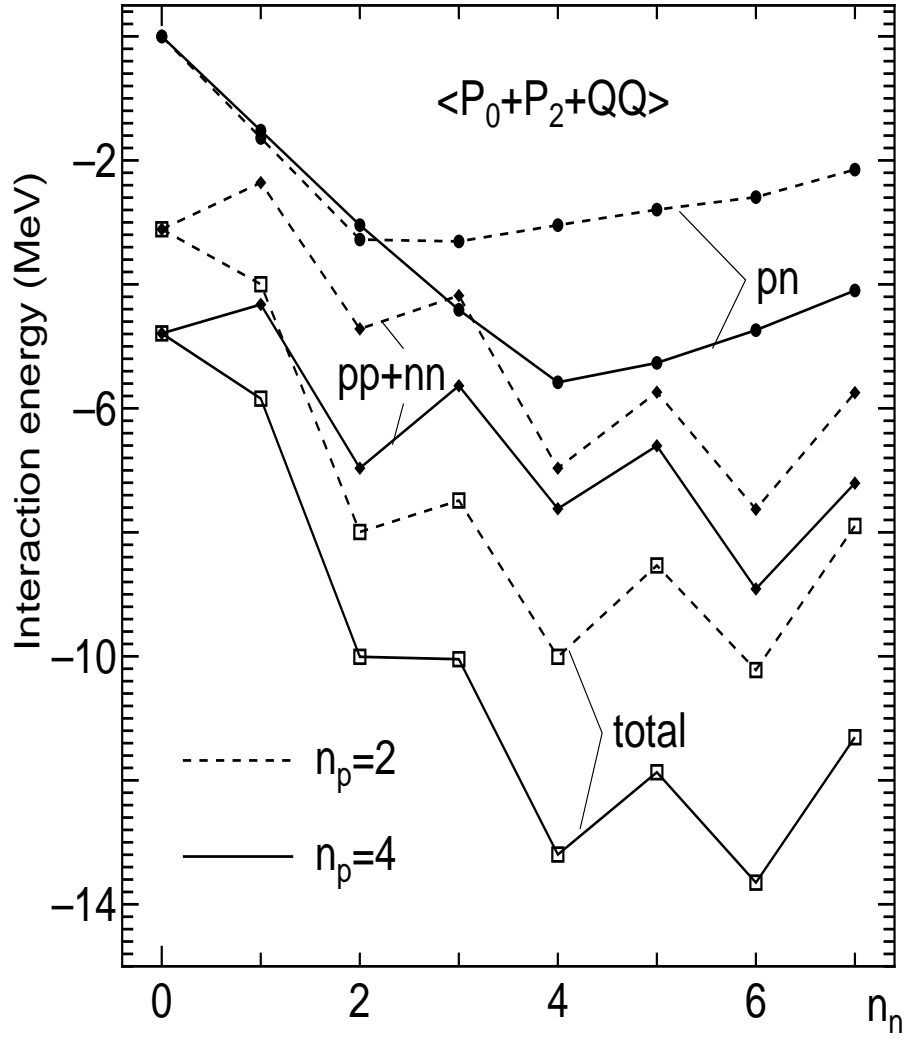


FIG. 2. p - n and p - p plus n - n components of interaction energy of the $P_0 + P_2 + QQ$ force in the $n_p=2$ and 4 isotopes.